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GATE 2016

Detailed Solutions For
Electronics & Communication Engg.

Date: 31-01-2016
Forenoon Session

Q.1 – Q.5 Carry one mark each

01. Based on the given statements, select the appropriate option with respect to grammar a statements.
    (i) The height of Mr. X is 6 feet.   (ii) The height of Mr. Y is 5 feet

    (A) Mr. X is longer than Mr. Y
    (B) Mr. X is more elongated than Mr. Y
    (C) Mr. X is taller than Mr. Y
    (D) Mr. X is lengthier than Mr. Y

01. Ans: (C)
Sol: In degrees of comparison Mr. X is taller than Mr. Y is apt.
    Positive degree   –   tall
    Comparative degree   –   taller
    Superlative degree   –   tallest

02. The students _____ the teachers on teachers day for twenty years of dedicated teaching.
    (A) facilitated   (B) felicitated   (C) fantasized   (D) facilitated

02. Ans: (B)
Sol: Felicitate means honour.

03. After India's cricket world cup victory in 1985, Shrotria who was playing both tennis and cricket till then, decided to concentrate only on cricket. And the <u>rest is history</u>.
    What does the underlined phrase mean in this context?
    (A) history will rest in peace   (B) rest is recorded in history books
    (C) rest is well known   (D) rest in archaic

03. Ans: (C)
Sol: 'rest is history' is an idiomatic expression which means 'rest is well known

04. Given $(9 \text{ inches})^{1/2} = (0.25 \text{ yards})^{1/2}$, which one of the following statements is TRUE?
    (A) 3 inches = 0.5 yards   (B) 9 inches = 1.5 yards
    (C) 9 inches = 0.25 yards   (D) 81 inches = 0.0625 yards

04. Ans: (C)
Sol: Given $(9 \text{ inches})^{1/2} = (0.25 \text{ yards})^{1/2}$
    9 inches = 0.25 yards

05. S, M, E and F are working in shifts in a term to finish a project. M works with twice the efficiency of other but for half as many days as E worked. S and M have 6 hour shifts in a day, whereas E and F have 12 hours shifts. What is the ratio of contribution of M to contribution of E in the project?
    (A) 1:1   (B) 1:2   (C) 1:4   (D) 2:1

05. Ans: (B)
Sol: M efficiency = 2 [efficiency of S,E, and F]
    Contribution of M in the project = x days × 6 hrs ×2
    Contribution of E in the project = 2x days × 12 hrs ×1
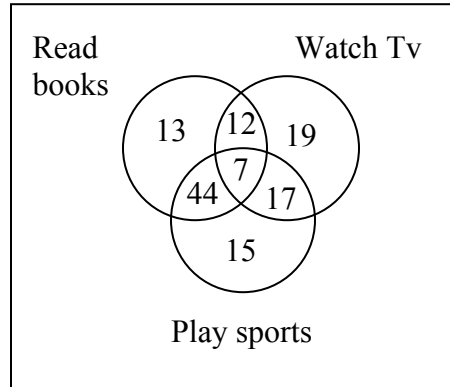   Contribution of M : Contribution of E
          x × 6 × 2 : 2x × 12 × 1
               1 : 2

Q.6 – Q.10 Carry two marks each

06. The Venn diagram shows the preference of the student population for leisure activities.

From the data given, the number of students who like to read books or play sports is _____.

(A) 44

(B) 51

(C) 79

(D) 108

06.   Ans: (D)

Sol: Read books = n(R) = 12 + 44 + 7 + 13

$\qquad$ = 76

Play sports = n(s) = 44 + 7 + 17 +15

$\qquad$ = 83

n (R $\cap$ S) = 44 + 7

$\qquad$ = 51

n (R $\cup$ S) = n (R) + n (S) − n (R $\cap$ S)

$\qquad$ = 76 + 83 − 51

$\qquad$ = 108

07. Social science disciplines were in existence in an amorphous form until the colonial period when they were institutionalized. In varying degrees, they were intended to further the colonial interest. In the time of globalization and the economic rise of postcolonical countries like India, conventional ways of knowledge production have become obsolete.

Which of the following can be logically inferred from the above statements?

(i) Social science disciplines have become obsolete.

(ii) Social science disciplines had a pre-colonial origin

(iii) Social science disciplines always promote colonialism

(iv) Social science must maintain disciplinary boundaries

(A) (ii) only

(B) (i) and (iii) only

(C) (ii) and (iv) only

(D) (iii) and (iv) only

07.   Ans: (A)

Sol: Until the colonial period means pre-colonial origin. Other options can't be inferred.

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08. Two and a quarter hours back, when seen in a mirror, the reflection of a wall clock without number markings seemed to show 1:30. What is the actual current time shown by the clock?
    (A) 8:15   (B) 11:15   (C) 12:15   (D) 12:45

08. Ans: (D)

Sol: Time back = $2\dfrac{1}{4}$ = 2 hrs 15 min

Clock time (C.T) + Mirror Time (M.T) = 12
$$\downarrow$$
$$1.30$$

$$\therefore \text{C.T} = \overset{\diagup 60}{12.00}$$
$$\underline{1.30}$$
$$\underline{10.30}$$

∴ The actual time shown by the clock = 10.30 + 2.15 = 12.45

09. M and N start from the same location. M travels 10 km East and then 10 km North-East. N travels 5 km South and then 4 km South-East. What is the shortest distance (in km) between M and N at the end of their travel?
    (A) 18.60   (B) 22.50   (C) 20.61   (D) 25.00

09. Ans: (C)

Sol: From the given data, the following diagram is possible

$$\cos 45^\circ = \frac{DE}{4}$$
$$DE = \cos 45^\circ \times 4$$
$$= 2.828 \text{ km}$$
$$\sin 45^\circ = \frac{EN}{4}$$
$$EN = \sin 45^\circ \times 4 = 2.828 \text{ km}$$

$\sin 45^\circ = \dfrac{EN}{4}$

$EN = \sin 45^\circ \times 4 = 2.828$ km

$CN = NE + CE = 2.828 + 5$

$\qquad\qquad = 7.828$ km

$CB = AB - AC = 10 - 2.828$

$\qquad\qquad = 7.171$ km

$(NB)^2 = (NC)^2 + (BC)^2$

$\qquad = (7.828)^2 + (7.171)^2$

$\therefore NB = \sqrt{(7.828)^2 + (7.171)^2} = 10.616$ km

$\therefore NM = NB + BN = 10.616 + 10 = 20.61$ km

10. A wire of length 340 mm is to be cut into two parts. One of the parts is to be made into a square and the other into a rectangle where sides are in the ratio of 1:2. What is the length of the side of the square (in mm) such that the combined area of the square and the rectangle is a MINIMUM?

(A) 30   (B) 40   (C) 120   (D) 180

10. Ans: (B)

Sol: Length of the wire = 340 m

Square   Rectangle

Perimeter of rectangle $= 2\left[\dfrac{x}{3} + \dfrac{2x}{3}\right]$

$\qquad\qquad\qquad\qquad = 2x$

Perimeter of square $= 340 - 2x$

Side of square $= \dfrac{340 - 2x}{4}$

Total area = Area of square + Area of rectangle

$\qquad = \left[\dfrac{340 - 2x}{4}\right]^2 + \dfrac{x}{3} \times \dfrac{2x}{3} = \left[\dfrac{340 - 2x}{4}\right]^2 + \dfrac{2x^2}{9}$

Combined area of square + rectangle = minimum

$f'(x) = 0$

$f(x) = \left[\dfrac{340 - 2x}{4}\right]^2 + \dfrac{2}{9}x^2$

$f'(x) = \dfrac{4}{9}x^2 - \dfrac{340 - 2x}{4} = 0$

$$\frac{4}{9}x^2 = \frac{1}{4}[340 - 2x]$$

$$= x = 90$$

Side of square $= \dfrac{340 - 2x}{4} = 40\,mm$

Another method:
Elimination procedure from alternatives option [C] and [D] are not possible because area may be maximum.

Option (A)
Side of the square = x = 30 mm
Perimeter of the square = 30 + 30 + 30 + 30 = 120 mm
∴ Perimeter of the rectangle = 340 – 120 = 220 mm
2x + 2 × 2x = 220
 x = 37
 2x = 37 ×2 = 74
Area of square = x^2 = $(30)^2$ = 900
Area of rectangle = x × 2x = 37 × 74 = 2738
Total area = 900 + 2738 = 3638 mm^2

Option (B)
Side of the square = x = 40 mm
Perimeter of the square = 340 – 160 = 180 mm
2x + 2 × 2x = 180 mm
 6x = 180 mm
 x = 30 mm
Area of the square = 40 × 40 = 1600 mm^2
Area of the rectangle = 30 × 2 × 30 = 1800 mm^2
∴ Total area = 1600 + 1800 = 3400 mm^2
∴ 3400 mm^2 < 3638 mm^2
Option B is correct.

Q.1 – Q.25 Carry one mark each.

01. The value of x for which the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 9 & 7 & 13 \\ -6 & -4 & -9+x \end{bmatrix}$

 has zero as an eigen value is _____

01. **Ans: x = 1**
Sol: For eigen value of A is to be zero, det (A) = 0
 3 {(−63 + 7x) + 52} −2{ (−81 + 9x)+78} + 4 { −36 + 42} = 0
 ∴ x = 1

02. Consider the complex valued function $f(z) = 2z^3 + b\,|z|^3$ where z is a complex variable. The value of b for which the function f(z) is analytic is _____

02. **Ans: 0**

Sol: $f(Z) = 2z^3 + b\,|z|^3$

for b = 0, f(z) becomes polynomial

so it is analytic every where only when b = 0

03. As x varies from – 1 to 3, which of the following describes the behaviour of the function $f(x) = x^3 - 3x^2 + 1$?

(A) f(x) increases monotonically

(B) f(x) increases, then decreases and increases again

(C) f(x) decreases, then increases and decreases again

(D) f(x) increases and then decreases

03. **Ans: (B)**

Sol: Since, f(−1) = −3, f(0) = 1, f(1) = −1,

f(2) = −3, f(3) = 1

04. How many distinct values of x satisfy the equation sin(x) = x/2, where x is in radians?

(A) 1   (B) 2   (C) 3   (D) 4 or more

04. **Ans: (C)**

Sol: $\sin x = \dfrac{x}{2}$ touches at 3 points.

05. Consider the time-varying vector I = in Cartesian coordinates, where $\omega > 0$ is a constant. When the vector magnitude $|I|$ is at its minimum value, the angle θ that I makes with the x axis (in degree, such that $0 \le \theta \le 180$) is _____

05. **Ans: 90°**

Sol:

If θ = 0

$|I| = 15$ $\left.\begin{array}{l} 0 < \theta < \dfrac{\pi}{2} \end{array}\right.$

If $\theta = \dfrac{\pi}{2}$ $15 \le \theta \le 5$

$|I| = 5$

$I = 15\cos\omega t\,\hat{a}_x + 5\sin\omega t\,\hat{a}_y$

06.   In the circuit shown below, V_s is a constant voltage source and I_L is a constant current load

The value of I_L that maximizes the power absorbed by the constant current load is

(A) $\dfrac{V_s}{4R}$ (B) $\dfrac{V_s}{2R}$ (C) $\dfrac{V_s}{R}$ (D) ∞

06. Ans: (B)

Sol: Maximum power delivered by the source to any load

$$\Rightarrow P_{max} = \frac{v_s^2}{4R}(w)$$
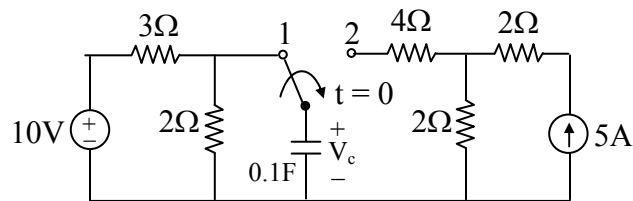
$\Rightarrow$ Here power absorbed by the load

$$P_L = v_L . I_L(W)$$
$$= (V_S - I_L . R) I_L$$
$$= V_S . I_L - I_L^2 . R (W)$$

If $I_L = \dfrac{V_S}{2R}$

$$\Rightarrow P_L = v_S . \frac{v_S}{2R} - \left(\frac{v_s}{2R}\right)^2 . R = \frac{v_s^2}{2R} - \frac{v_s^2}{4R} \Rightarrow \frac{v_s^2}{4R}(w)$$

$$P_L = P_{max}$$

07.   The switch has been in position 1 for a long time and abruptly changes to position 2 at t = 0

If time t is in seconds, the capacitor voltage V_C (in volts) for t > 0 given by

(A) 4 (1 – exp (–t /0.5))   (B) 10 – 6exp (–t/0.5))

(C) 4 (1–exp (t/0.6))   (D) 10 – 6 exp (–t /0.6)

07. Ans: (D)

Sol: $v_c(0^-) = \left(\dfrac{10}{5}\right).2 = 4v = v_c(0^+)$

$v_c(\infty) = 5 \times 2 = 10v$

$\tau = Req.C = 6 \times 0.1 = 0.6\,sec$

$V_c(t) = 10 + (4 - 10)e^{-t/T} = 10 - 6e^{-\frac{t}{0.6}} v \ for\ 0 \le t \le \infty$

08. The figure shows an RLC circuit with a sinusoidal current source.

At resonance, the ratio $|I_L|/|I_R|$, i.e., the ratio of the magnitudes of the inductor current phasor and the resistor current phasor, is _____

08.   Ans: 0.3163

Sol: At resonance, $I_R = I$

$$I_L = QI\angle -90^0\, A$$

$$I_C = QI\angle 90^o A$$

Where $Q = W_0 CR$

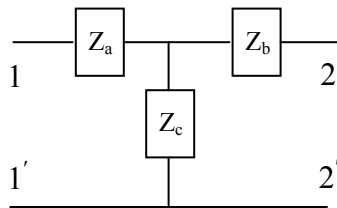$$= R\sqrt{\frac{C}{L}} = 10\sqrt{\frac{10\times 10^{-6}}{10\times 10^{-3}}}$$

$$= \frac{1}{\sqrt{10}} = 0.3163$$

So, $\dfrac{|I_L|}{|I_R|} = Q = 0.3163$

09. The z-parameter matrix for the two-port network shown is

$$\begin{bmatrix} 2j\omega & j\omega \\ j\omega & 3+2j\omega \end{bmatrix}$$

Where the entries are is Ω. Suppose $Z_b(j\omega) = R_b + j\omega$

Then the value of R_b (in Ω) equals _____

09.   Ans: 3

Sol: $z(s) = \begin{bmatrix} 2s & s \\ s & 3+2s \end{bmatrix} = \begin{bmatrix} z_A + z_C & z_C \\ z_C & z_B + z_C \end{bmatrix}$

Here, $z_c = s$

And $z_B + z_C = 3 + 2s$

$\Rightarrow z_B + s = 3 + 2s$

$\Rightarrow z_B = 3+s \Rightarrow R_B + jx_B = 3 + j\omega \Rightarrow R_B = 3\Omega$

10. The energy of the signal x(t) = $\dfrac{\sin(4\pi t)}{4\pi t}$ is _____

10. **Ans: 0.25**

Sol: $\dfrac{\sin at}{\pi} \leftrightarrow rect\left(\dfrac{\omega}{2a}\right)$

$X(\omega) = \dfrac{1}{4} rect\left(\dfrac{\omega}{8\pi}\right)$

$E_{x(t)} = \dfrac{1}{2\pi} \int\limits_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega$

$\qquad = \dfrac{1}{2\pi} \times \dfrac{1}{16} \times 8\pi$

$\qquad = \dfrac{1}{4} = 0.25$

11. The Ebers – Moll model of a BJT is valid
    (A) only in active mode
    (B) only in active and saturation modes
    (C) only in active and cut-off modes
    (D) in active, saturation and cut-off modes

11. **Ans: (A)**

12. A long-channel NMOS transistor is biased in the linear region $V_{DS} = 50$ mV and is used as a resistance. Which one of the following statements is NOT correct?
    (A) If the device width W is increased, the resistance decreases
    (B) If the threshold voltage is reduced, the resistance decrease
    (C) If the device length L is increased, the resistance increases
    (D) If V_{GS} is increased, the resistance increases

12. **Ans: (D)**

Sol: A. TRUE
    B. TRUE
    C. TRUE
    D. FALSE

$r_{ds}(on) = \dfrac{1}{\mu_n cox \dfrac{W}{L}\left[V_{gs} - V_T\right]}$

13. Assume that the diode in the figure has $V_{on} = 0.7$ V, but is otherwise ideal.
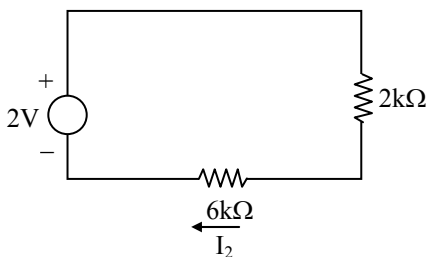

The magnitude of the current i_2 (in mA) is equal to _____
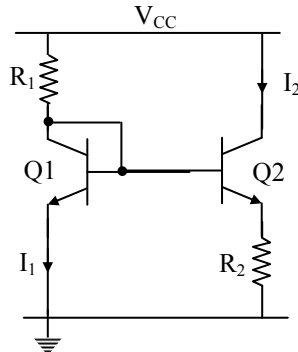
13. Ans: 0.25

Sol:

Diode needs at least 0.7V, with 0.5V at the terminals, the diode is OFF. Therefore the circuit reduces to

$$I_2 = \frac{2}{2k + 6k}$$

$$= \frac{2}{8k}$$

$$= 0.25\,\text{mA}$$

14. Resistor R_1 in the circuit below has been adjusted so that $I_1 = 1$ mA. The bipolar transistor Q1 and Q2 are perfectly matched and have very high current gain, so their base currents are negligible. The supply voltage V_{cc} is 6 V. The thermal voltage kT/q is 26 mV.

The value of R_2 (in Ω) for which $I_2 = 100$ μA is _____

14.　Ans: 598.67

Sol: $I_C = I_S e^{\frac{V_{be}}{V_t}}$

$$\therefore V_{be1} = V_t \ell n\left[\frac{I_c}{I_s}\right] = V_t \ell n\left[\frac{I_1}{I_s}\right]$$

$$V_{be2} = V_t \ell n\left[\frac{I_2}{I_s}\right]$$

From the circuit

$$V_{be1} = V_{be2} + I_2 R_2 \text{ and } V_t = \frac{kT}{q} = 26mV$$

$$\therefore R_2 = \frac{V_{be1} - V_{be2}}{I_2} = \frac{V_t \ell n\left[\frac{I_1}{I_2}\right]}{I_2} = \frac{26mV \, \ell n\left[\frac{1mA}{100\mu A}\right]}{100 \, \mu A} = 598.67\Omega$$

15. Which one of the following statements is correct about an ac-coupled common-emitter amplifier operating in the mid-band region?
    (A) The device parasitic capacitances behave like open circuits, whereas coupling and bypass capacitances behave like short circuits.
    (B) The device parasitic capacitances, coupling capacitances and bypass capacitances behave like open circuits.
    (C) The device parasitic capacitances, coupling capacitances and bypass capacitances behave like short circuits.
    (D) The device parasitic capacitances behave like short circuits, whereas coupling and bypass capacitances behave like open circuits.

15.　Ans: (A)

Sol: The parasitic capacitances are in PF and the coupling and bypass capacitors are in μF. Therefore for the mid frequency band, parasitic capacitance act like open circuits and coupling and bypass capacitances act like short circuits.

16. Transistor geometries in a CMOS inverter have been adjusted to meet the requirement for worst case charge and discharge times for driving a load capacitor C. This design is to be converted to that of a NOR circuit in the same technology, so that its worst case charge and discharge times while driving the same capacitor are similar. The channel length of all transistors are to be kept unchanged. Which one of the following statements is correct?
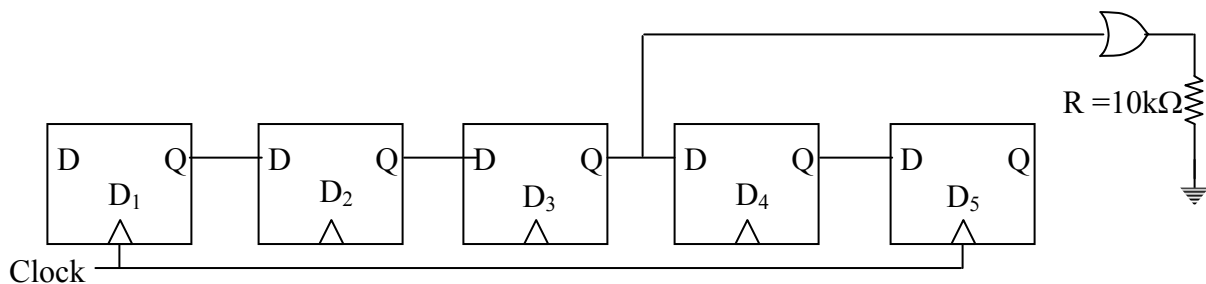

(A) Widths of PMOS transistors should be doubled, while widths of NMOS transistors should be halved.

(B) Widths of PMOS transistors should be doubled, while widths of NMOS transistors should not be changed.

(C) Widths of PMOS transistors should be halved, while widths of NMOS transistors should not be changed.

(D) Widths of PMOS transistors should be unchanged, while widths of NMOS transistors should be halved.

16. **Ans: (B)**

Sol: Width of PMOS transistors should be halved. while width of NMOS transistors should not be changed, because NMOS transistors are in parallel. If any one transistor ON, output goes to LOW.

17. Assume that all the digital gates in the circuit shown in the figure are ideal, the resistor R = 10 kΩ and the supply voltage is 5V. The D flip-flops D_1, D_2 D_3, D_4 and D_5 are initialized with logic values, 0, 1, 0, 1 and 0, respectively. The clock has a 30% duty cycle.

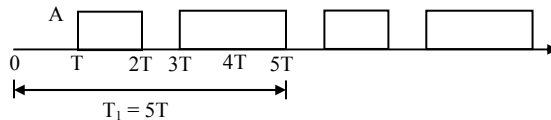The average power dissipated (in mW) in the resistor R is _____

17.   Ans: 1.5

Sol:

Clk	Q_1	Q_2	Q_3	Q_4	Q_5	$Y = Q_3 + Q_5$
0	0	1	0	1	0	0
1	0	0	1	0	1	1
2	1	0	0	1	0	0
3	0	1	0	0	1	1
4	1	0	1	0	0	1
5	0	1	0	1	0	0

The waveform at OR gate output, Y is [A = +5V]

Average power

$$P = \frac{V_{Ao}^2}{R} = \frac{1}{R}\left[\underset{T_1 \to \infty}{Lt}\frac{1}{T_1}\int_{o}^{T_1} y^2(t)\,dt\right]$$

$$P = \frac{1}{RT_1}\left[\int_{T}^{2T} A^2 dt + \int_{3T}^{5T} A^2 dt\right] = \frac{A^2}{RT_1}\left[(2T - T) + (5T - 3T)\right] = \frac{A^2.3T}{R(5T)} = \frac{5^2.3}{10 \times 5} = 1.5\,mw$$

18.   A 4:1 multiplexer is to be used for generating the output carry of a full adder. A and B are the bits to be added while C_{in} is the input carry and C_{out} is the output carry. A and B are to be used as the select bits with A being the more significant select bit.

Which one of the following statements correctly describes the choice of signals to be connected to the inputs I_0, I_1, I_2 and I_3 so that the output is C_{out}?

(A) $I_0 = 0$, $I_1 = C_{in}$, $I_2 = C_{in}$ and $I_3 = 1$ (B) $I_0 = 1$, $I_1 = C_{in}$, $I_2 = C_{in}$ and $I_3 = 1$

(C) $I_0 = C_{in}$, $I_1 = 0$, $I_2 = 1$ and $I_3 = C_{in}$ (D) $I_0 = 0$, $I_1 = C_{in}$, $I_2 = 2$ and $I_3 = C_{in}$

18.   Ans: (A)

Sol: C_{i+1} (A, B, C_i) = Σm (3, 5, 6, 7) using 4:1 max

	I_0	I_1	I_2	I_3
$\overline{C}_{in}$	0	2	4	⑥
C_{in}	1	③	⑤	⑦
	0	C_{in}	C_{in}	1

19. The response of the system $G(s) = \dfrac{s-2}{(s+1)(s+3)}$ to the unit step input u(t) is y(t).

The value of $\dfrac{dy}{dt}$ at t=0$^+$ is _____

19. **Ans: 1**

Sol: Method 1: Given $Y(s) = \dfrac{(s-2)}{(s+1)(s+3)} u(s)$

$\Rightarrow Y(S) = \dfrac{(s-2)}{s(s+1)(s+3)}$ [Given $u(s) = \dfrac{1}{s}$]

$L\left[\dfrac{dy}{dt}\right] = sY(s)$

$sY(s) = \dfrac{(s-2)}{(s+1)(s+3)}$

$\left.\dfrac{dy}{dt}\right|_{t=0^+} = \underset{s\to\infty}{Lt}\left(\dfrac{(s-2)}{(s+1)(s+3)}\right) = 1$

Method: 2

$Y(s) = \left(\dfrac{(s-2)}{s(s+1)(s+3)}\right) = \dfrac{-2}{3s} + \dfrac{3}{2(s+1)} - \dfrac{5}{6(s+3)}$

$y(t) = -2/3 + 3/2\ e^{-t} - 5/6e^{-3t}$

$\dfrac{dy}{dt}(t = 0+) = 3/2(-1)e^{-t} - \dfrac{5}{6}(-3)e^{-3t} = -3/2 + 5/2 = 1$

20. The number and direction of encirclements around the point $-1 + j0$ in the complex plane by the Nyquist plot of $G(S) = \dfrac{1-s}{4+2s}$ is

(A) zero   (B) one, anti-clockwise   (C) one, clockwise   (D) two, clockwise

20. **Ans: (A)**

Sol:

Number of Encirclements about $(-1, j0)$ is Zero

21. A discrete memoryless source has an alphabet (a_1, a_2, a_3, a_4) with corresponding probabilities $\left(\dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{8}\right)$. The minimum required average codeword length in bits to represent this source for error-free reconstruction is_____

21. **Ans: 1.75**

Sol: $H = \dfrac{1}{2}\log_2 2 + \dfrac{1}{4}\log_2 4 + \dfrac{1}{8}\log_2 8 + \dfrac{1}{8}\log_2 8$ ($\because$ $L_{min} = H$)

$= 1.75$ bits/ word

22. A speech signal is sampled at 8 kHz and encoded into PCM format using 8 bits/sample. The PCM data is transmitted through a baseband channel via 4-level PAM. The minimum bandwidth (in kHz) required for transmission is_____

22. **Ans: 16**

Sol: $R_b = 64$ kbps

BW = 32 kHz

$(B_T)_{min} = \dfrac{R_b}{2\log_2 M} = \dfrac{R_b}{2\log_2 4} = \dfrac{R_b}{2 \times 2} = \dfrac{R_b}{4} = 16\,kHz$

23. A uniform and constant magnetic field $B = \hat{z}B$ exists in the $\hat{z}$ direction in vacuum. A particle of mass m with a small charge q is introduced into this region with an initial velocity $v = \hat{x}v_x + \hat{z}v_2$. Given that B, m, q, v_x and v_z are all non-zero, which one of the following describes the eventual trajectory of the particle?

(A) Helical motion in the $\hat{z}$ direction
(B) Circular motion in the xy plane
(C) Linear motion in the $\hat{z}$ direction
(D) Linear motion in the $\hat{x}$ direction

23. Ans: (A)

Sol: Given $\vec{B} = B\hat{Z}$

$$\vec{V} = V_X\hat{X} + V_Z\hat{Z}$$

x – component of $\vec{V}$ is perpendicular to magnetic field $\vec{B}$.

A change moving perpendicular to the magnetic field experience a radial force causing circular motion shown in figure.

z- component of $\vec{V}$ is parallel to magnetic field $\vec{B}$. A change moving parallel to the field generates no force shown in figure (b).

$\therefore$ Motion with components perpendicular and parallel to the field causes the change to move in a helical path along +z direction. Show in figure (c).

Fig (a)   Fig (b)   Fig (c)

24. Let the electric field vector of a plane electromagnetic wave propagating in a homogenous medium be expressed as $E = \hat{x}E_x e^{-j(wt-\beta z)}$, where the propagation constant β is a function of the angular frequency ω. Assume that $\beta(\omega)$ and E_x are known and are real. From the information available, which one of the following CANNOT be determined?

(A) The type of polarization of the wave
(B) The group velocity of the wave
(C) The phase velocity of the wave
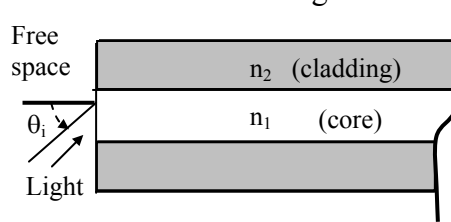(D) The power flux through the z = 0 plane

24. Ans: (D)

Sol: Given $\vec{E} = \hat{x}E_x e^{-j(\omega t-\beta z)}$

As medium properties and area of z = 0 plane is not given in the data, hence Average power flow (or) power flux cannot be determined.

25. Light from the free space is incident at an angle θ_I to the normal of the facet of a step-index large core optical fibre. Th core and cladding refractive indices are $n_1 = 1.5$ and $n_2 = 1.4$, respectively.

Free space

n_2 (cladding)

θ_i n_1 (core)

Light

The maximum value of θ_i (in degrees) for which the incident light will be guided in the core of the fibre is_____

25. Ans: 32.58

Sol: Given $n_1 = 1.5$, $n_2 = 1.4$

The maximum angle over which the incident light rays entering the fiber is called acceptance angle, θ_A.

$$\sin\theta_A = \sqrt{n_1^2 - n_2^2}$$

(or) $\theta_A = \sin^{-1}\sqrt{n_1^2 - n_2^2} = \sin^{-1}\sqrt{1.5^2 - 1.4^2} \Rightarrow \theta_A = 32.58°$

Q. 26 – Q. 55 carry two marks each

26. The ordinary differential equation $\dfrac{dx}{dt} = -3x + 2$, with $x(0) = 1$ is to be solved using the forward Euler method. The largest time step that can be used to solve the equation without making the numerical solution unstable is_____

26.

Sol: $\dfrac{dy}{dx} = -3y + 2$, $y(0) = 1$

If $|1 - 3h| < 1$ then solution of differential equation is stable

$\Rightarrow -1 < 1 < -3h < 1$

$\Rightarrow -2 < -3h < 0$

$\Rightarrow 0 < 3h < 2$

$\Rightarrow 0 < h < \dfrac{2}{3}$

$\therefore$ If $0 < h < 0.66$ then we get stable

27. Suppose C is the closed curve defined as the circle $x^2 + y^2 = 1$ with C oriented anti-clockwise. The value of $\oint(xy2dx + x2ydy)$ over the curve C equals____

27. Ans: 0 (Zero)

Sol: Using Green's Theorem

$$\oint_C (xy^2dx + x^2ydy) = \iint_R (2xy - 2xy)dxdy = 0$$

28. Two random variables X and Y are distributed according to

$$f_{X,Y}(x, y) = \begin{cases} (x+y), & 0 \le x \le 1, \quad 0 \le y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$
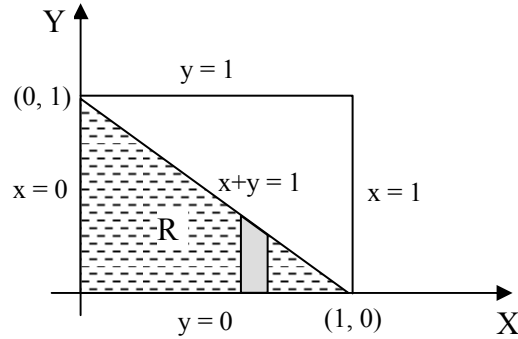
The probability $P(X + Y \le 1)$ is _____

28. Ans: 0.33

Sol: $P(X + Y \le 1) = \int\limits_R f(x, y)dxdy$

$$= \int\limits_{x=0}^{1} \int\limits_{y=0}^{1-x} (x + y)dxdy$$

$$= \int\limits_{0}^{1} \left(xy + \frac{y^2}{2} \right)_{0}^{1-x} dx$$

$$= \int\limits_{0}^{1} \left[x(1-x) + \frac{(1-x)^2}{2} \right]dx$$

$$= 0.33$$

29. The matrix $A = \begin{bmatrix} a & 0 & 3 & 7 \\ 2 & 5 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & b \end{bmatrix}$ has det (A) = 100 and trace (A) = 14.

The value of $|a - b|$ is _____ .

29. Ans: 3

Sol: trace (A) = 14

a + b + 7 = 14

a + b = 7

det (A) = 100

$$5 \begin{vmatrix} a & 3 & 7 \\ 0 & 2 & 4 \\ 0 & 0 & b \end{vmatrix} = 100$$

10 ab = 100 $\Rightarrow$ ab = 10

$\therefore$ a = 5, b = 2 (or) a = 2, b = 5

$\Rightarrow |a - b| = 3$

30. In the given circuit, each resistor has a value equal to 1 Ω.

What is the equivalent resistance across the terminals a and b?

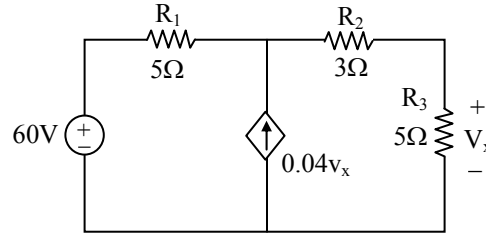(A) 1/6 Ω   (B) 1/3 Ω   (C) 9/20 Ω   (D) 8/15 Ω

30. Ans: (D)

Sol:

$\Rightarrow$

$\Rightarrow$

so, $R_{ab} = \dfrac{12}{105} + \left(\dfrac{240}{1155} + \dfrac{4}{11}\right) 11\left(1 + \dfrac{60}{105}\right)$

$\qquad = 0.1143 + 0.41485$

$\qquad = 0.53\Omega = \dfrac{8}{15}\Omega$

31. In the circuit shown in the figure, the magnitude of the current (in amperes) through R_2 is _____

31. Ans: 5

Sol: Nodal $\Rightarrow \dfrac{v - 60}{5} - 0.04v_x + \dfrac{v}{8} = 0$

Where $v_x = \dfrac{5v}{8}$

$\Rightarrow v = 40v$

So, $IR_2 = I_{3\Omega} = \dfrac{V}{8} = \dfrac{40}{8}A = 5A$

32. A continuous-time filter with transfer function $H(S) = \dfrac{2s + 6}{s^2 + 6s + 8}$ is converted to a discrete time

filter with transfer function $G(Z) = \dfrac{2z^2 - 0.5032\,z}{z^2 - 0.5032\,z + k}$ so that the impulse response of the

continuous-time filter, sampled at 2 Hz, is identical at the sampling instants to the impulse response of the discrete time filter, The value of k is _____.

32. Ans: 0.049

Sol: $H(s) = \dfrac{2s + 6}{s^2 + 6s + 8} = \dfrac{2s + 6}{(s + 2)(s + 4)} = \dfrac{1}{s + 2} + \dfrac{1}{s + 4}$

$h(t) = e^{-2t}u(t) + e^{-4t}.u(t)$

$T_s = \dfrac{1}{F_s} = \dfrac{1}{2}$

$h(nT_s) = e^{-2nTs}u(nTs) + e^{-4nTs}, u(nTs)$

$\qquad = e^{-n}u(n) + e^{-2n}.u(n)$

$H(z) = \dfrac{z}{z - e^{-1}} + \dfrac{z}{z - e^{-2}} = \dfrac{z}{z - 0.367} + \dfrac{z}{z - 0.135}$

$$H(z) = \frac{z^2 - 0.135z + z^2 - 0.367z}{z^2 - 0.5032z + 0.049}$$

$$= \frac{2z^2 - 0.5032z}{z^2 - 0.5032z + 0.049}$$

k = 0.049

33. The Discrete Fourier Transform (DFT) of the 4-point sequence
X[n] = {x[0], x[1], x[2], x[3]} = {3, 2, 3, 4} is
X[k] = {X[0], X[1], X[2], X[3]} = {12, 2j, 0, –2j}
If $X_1[k]$ is the DFT of the 12-point sequence $x_1[n]$ = {3, 0, 0, 2, 0, 0, 3, 0, 0, 4, 0, 0},

The value of $\left| \dfrac{x_1[8]}{X_1[11]} \right|$ is _____

33. **Ans: 6**

Sol: Interpolation in time domain equal to replication in frequency domain.

$$x_1(n) = x\left(\frac{n}{3}\right)$$

$X_1(k)$ = [12, 2j, 0, –2j, 12, 2j, 0, -2j, 12, 2j, 0, –2j]
$X_1(8) = 12$, $X_1(11) = -2j$

$$\left| \frac{X_1(8)}{X_1(11)} \right| = \left| \frac{12}{-2j} \right| = 6$$

34. The switch S in the circuit shown has been closed for a long time. It is opened at time t = 0 and remains open after that. Assume that the diode has zero reverse current and zero forward voltage drop.
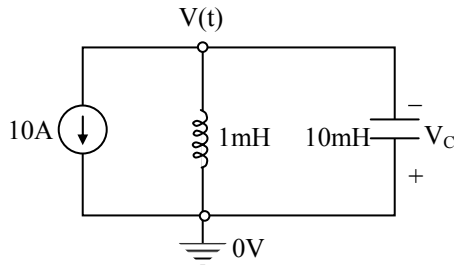

The steady state magnitude of the capacitor voltage V_C (in volts) is _____

34. Ans: 100
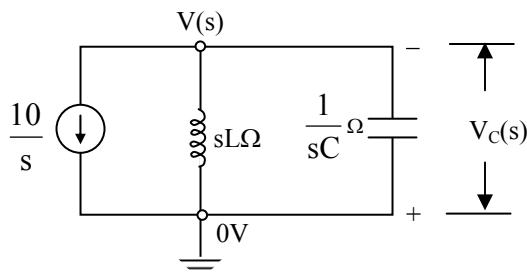
Sol: $i_L(0^-) = \dfrac{10}{1} = 10A = i_L(0^+)$

$v_c(0^-) = 0v = v_c(0^+)$

For diode, $R_r = \infty\Omega$ and $R_f = 0\Omega$ (given)

For t ≥ 0

Transform the above network in Laplace domain

Nodal $\Rightarrow \dfrac{10}{5} + \dfrac{v(s)}{sL} + \dfrac{v(s)}{\dfrac{1}{sc}} = 0$

$\Rightarrow v(s) = \dfrac{-10L}{s^2 LC + 1}$

$= v(s) = -10L.\dfrac{1}{\sqrt{LC}}.\dfrac{\dfrac{1}{\sqrt{LC}}}{s^2 + \dfrac{1}{LC}}$

$$= -10 \left| \sqrt{\frac{L}{C}} \cdot \frac{\omega_n}{s^2 + \omega_n^2} \right| \text{ where } \omega_n^2 = \frac{1}{LC}$$
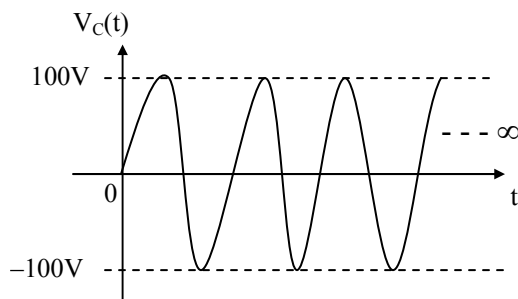
$$\Rightarrow v(t) = -10\sqrt{\frac{L}{C}} \sin \omega_n t \quad \text{for } 0 \le t \le \infty$$

Where $\omega_n = \dfrac{1}{\sqrt{LC}} \text{rad/sec}$

$$\Rightarrow v(t) = -100 \sin 10^4 t \quad \text{for } 0 \le t \le \infty$$

By KVL $\Rightarrow v(t) + v_c(t) = 0$

$$\Rightarrow v_c(t) = -v(t)$$
$$= 100 \sin(10000t) \text{ V for } 0 \le t \le \infty$$

So, $v_c = 100\angle{-90}v$

$$\Rightarrow |v_c| = 100v$$

35. A voltage V_G is applied across a MOS capacitor with metal gate and p-type silicon substrate at T = 300 K. The inversion carrier density (in number of carriers per unit area) for $V_G = 0.8$ V is 2×10^{11} cm^{-2}. For $V_G = 1.3$ V, the inversion carrier density is 4×10^{11} cm^{-2}. What is the value of the inversion carrier density for $V_G = 1.8$ V?

    (A) 4.5×10^{11} cm^{-2} (B) 6.0×10^{11} cm^{-2} (C) 7.2×10^{11} cm^{-2} (D) 8.4×10^{11} cm^{-2}

35. Ans: (B)

36. Consider avalanche breakdown in a silicon p$^+$n junction. The n-region is uniformly doped with a donor density N_D. Assume that breakdown occurs when the magnitude of the electric field at any point in the device becomes equal to the critical filed E_{crit}. Assume E_{crit} to be independent of N_D. If the built-in voltage of the p$^+$n junction is much smaller than the breakdown voltage, V_{BR}, the relationship between V_{BR} and N_D is given by

    (A) $V_{BR} \times \sqrt{N_D}$ = constant   (B) $N_D \times \sqrt{V_{BR}}$ = constant

    (C) $N_D \times V_{BR}$ = constant   (D) N_D / V_{BR} = constant

36. Ans: (C)

Sol: $\cancel{V_0} + V_{BR} = \dfrac{\in E_{CRIT}^2}{2q} \left[\dfrac{1}{N_D} + \dfrac{1}{\cancel{N_A}} \right]$

$$\left[P + N \to N_A \gg N_D \to \frac{1}{N_A} \ll \frac{1}{N_D} \right]$$

$V_{BR}.N_D = $ CONSTNAT

$\because E_{CRIT}$ is CONSTANT

37. Consider a region of silicon devoid of electrons and holes, with an ionized donor density of $N_d^+ = 10^{17}\,cm^{-3}$. The electric filed at x = 0 is 0 V/cm and the electric filed at x = L is 50 kV/cm in the positive x direction. Assume that the electric filed is zero in the y and z directions at all points.

$N_d^+ = 10^{17}\,cm^{-3}$

X = 0   X = L

Given q = 1.6 × 10^{-19} coulomb, $\in_0$ = 8.85 × 10^{-14} F/cm, $\in_r$ = 11.7 for silicon, the value of L in nm is _____.

37. Ans: [32.358 nm]

Sol: $E = \dfrac{eN_D}{\varepsilon}x \Rightarrow \dfrac{dE}{dx} = \dfrac{eN_D}{\varepsilon}$

$= \left[\dfrac{50kv/cm - 0}{L - 0} \right] = \dfrac{1.6 \times 10^{-19} \times 10^{17}}{11.7 \times 8.85 \times 10^{-14}}$

$\Rightarrow L = 3.2358 \times 10^{-6}\,cm$

$\quad\quad = 3.2358 \times 10^{-8}\,m$

$\quad\quad = 32.358 \times 10^{-9}\,m$

$\quad\quad = 32.358\,nm$

38. Consider a long-channel NMOS transistor with source and body connected together. Assume that the electron mobility is independent of V_{GS} and V_{DS}. Given,

$g_m = 0.5\,\mu A/V$ for $V_{DS} = 50\,mV$ and $V_{GS} = 2V$,

$g_d = 8\,\mu A/V$ for $V_{GS} = 2\,V$ and $V_{DS} = 0\,V$,

Where $g_m = \dfrac{\partial I_D}{\partial V_{GS}}$ and $g_d = \dfrac{\partial I_D}{\partial V_{DS}}$

The threshold voltage (in volts) of the transistor is

38. Ans: 1.2 Volts

Sol: $I_o = \mu_n c_{ox} \dfrac{W}{L}\left[(V_{gs} - V_T)V_{DS} - \dfrac{1}{2}V_{DS}^2 \right]$

$g_m = \dfrac{dI_D}{dV_{gs}} = \mu_n C_{ox} \dfrac{W}{L}.V_{DS}$

$$\Rightarrow \mu_n C_{ox} \frac{W}{L} = \frac{g_m}{V_{DS}} = \frac{0.5 \times 10^{-6}}{50 \times 10^{-3}} = 10 \times 10^{-6}$$

$$g_d = \frac{dI_D}{dV_{DS}} = \mu_n C_{ox} \frac{W}{L}\left[V_{gs} - V_T\right]$$

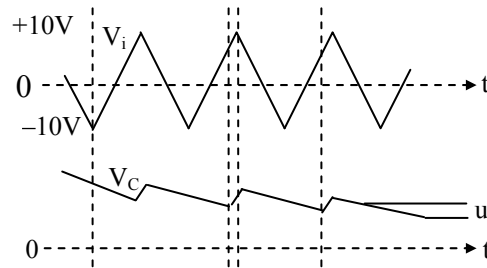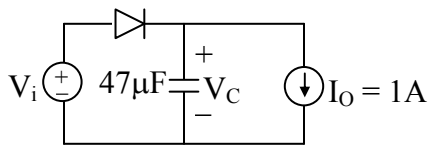$$8 \times 10^{-6} = 10 \times 10^{-6}\left[V_{gs} - V_T\right]$$

$$V_{gs} - V_T = \frac{8 \times 10^{-6}}{10 \times 10^{-6}}$$

$$\Rightarrow V_T = V_{gs} - 0.8$$
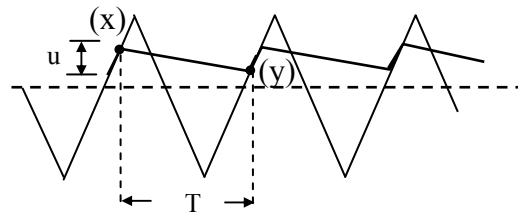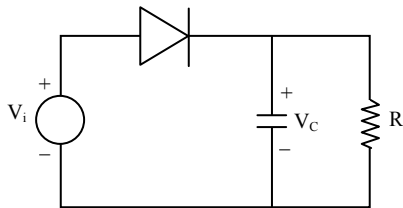
$$V_T = 2V - 0.8V = 1.2\ V$$

$$V_T = 1.2V$$

39. The figure shows a half-wave rectifier with a 475 µF filter capacitor. The load draws a constant current $I_O = 1$ A from the rectifier. The figure also shows the input voltage V_t, the output voltage V_C and the peak-to-peak voltage ripple u on V_C. The input voltage V_t is a triangle-wave with an amplitude of 10 V and a period of 1 ms.

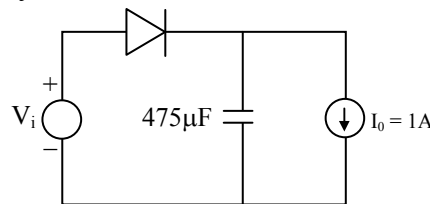The value of the ripple u (in volts) is _____.

39. Ans: 2.1V

Sol:

$$u = (V_{ipeak} - V_D) - (V_{ipeak} - V_D)\left[1 - \frac{T}{RC}\right]$$

Peak to peak Amplitude of ripple

$$u = \frac{V_{ipeak} - V_D}{RC}.T$$
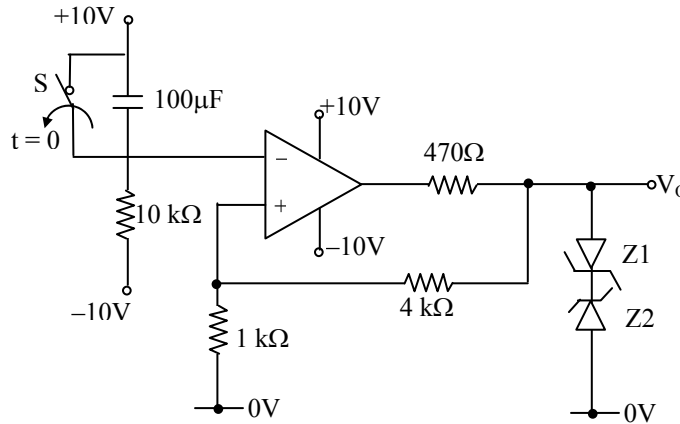
If the load is represented by a constant current source $\dfrac{V_{ipeak} - V_D}{R} = I_0$

$$\therefore u = \frac{I_0.T}{C} = \frac{1A.(1m)}{475\mu F}$$
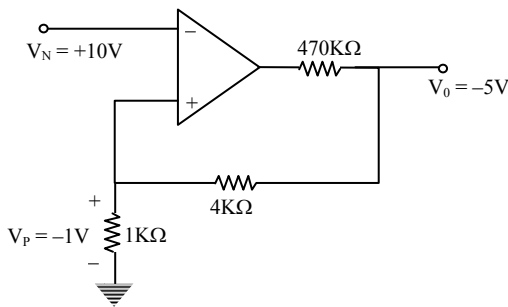
$$= 2.1V$$

40. In the opamp circuit shown, the Zener diodes Z1 and Z2 clamp the output voltage V_O to +5 V or −5 V. The switch S is initially closed and is opened at time t = 0
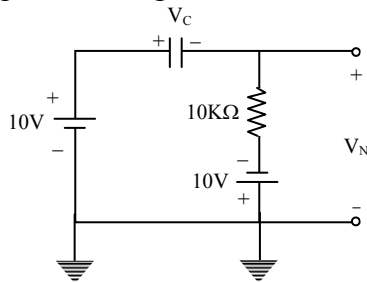

The time t = t_1 (in seconds) at which V_O changes state is _____.

40. Ans: 0.798 sec

Sol: At t = 0^-

The output V_0 changes state when V_N = 1V for t ≥ 0

$$V_C(t) = [V_C(0) - V_C(\infty)]e^{-t/RC} + V_C(\infty) = [0 - 20]e^{-t/10^4 \cdot 10^{-4}} + 20 = 20 - 20e^{-t}$$

$$V_N = 10 - V_c = 10 - [20 - 20e^{-t}] = -10 + 20e^{-t}$$

For op-amp to change state

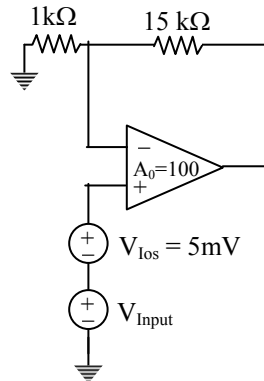$V_N = V_P$

$-10 + 20e^{-t} = -1$

$20e^{-t} = 9$

$$-t = \ln\left[\frac{9}{20}\right]$$

→t = 0.798sec

41. An opamp has a finite open loop voltage gain of 100. Its input offset voltage V_{ios} ($= +5mV$) is modeled as shown in the circuit below. The amplifier is ideal in all other respects. V_{input} is 25 mV.

The output voltage (in millivolts) is _____.

41. Ans: 413.79

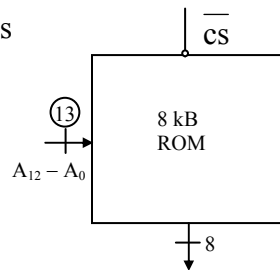**Sol:** $V_0 = \left[\dfrac{A}{1+A\beta}\right]\left[V_{ios} + V_{input}\right]$

$V_0 = \dfrac{100}{1+100\left[\dfrac{1k}{15k+1k}\right]}[25mV + 5mV] = \dfrac{1600}{116}[30mV] = 413.79mV$

42. An 8 Kbyte ROM with an active low Chip Select input ($\overline{CS}$) is to be used in an 8085 microprocessor based system. The ROM should occupy the address range 1000H to 2FFFH. The address lines are designated as A_{15} to A_0, where A_{15} is the most significant address bit. Which one of the following logic expression will generate the correct $\overline{CS}$ signal for this ROM?

(A) $A_{15} + A_{14} + (A_{13}.A_{12} + \overline{A_{13}}.\overline{A_{12}})$ (B) $A_{15}.A_{14}.(A_{13} + A_{12})$

(C) $\overline{A_{15}}.\overline{A_{14}}.(A_{13}.\overline{A_{12}} + \overline{A_{13}}.A_{12}$ (D) $\overline{A_{15}} + \overline{A_{14}} + A_{13}.A_{12}$

42. Ans: (A)

Sol: Address Range given is

A_{15}	A_{14}	A_{13}	A_{12}		A_{11}	A_{10}	A_9	A_8		A_7	A_6	A_5	A_4		A_3	A_2	A_1	A_0
1000H →	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0

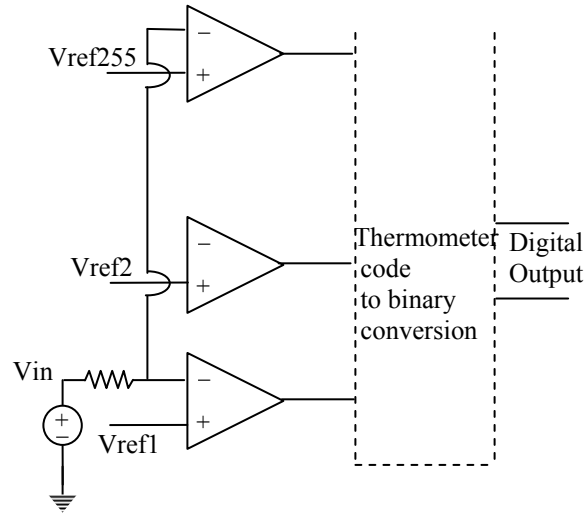|	A_{15}	A_{14}	A_{13}	A_{12}	A_{11}	A_{10}	A_9	A_8	A_7	A_6	A_5	A_4	A_3	A_2	A_1	A_0
1000H →	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
2FFFH →	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1

To provide $\overline{cs}$ as low, The condition is

$A_{15} = A_{14} = 0$ and $A_{13} A_{12} = 01$ (or) (10)

i.e $A_{15} = A_{14} = 0$ and $A_{13} A_{12}$ shouldn't be 00, 11.

Thus it is $A_{15} + A_{14} + [A_{13}A_{12} + \overline{A_{13}}.\overline{A_{12}}]$

43. In an N bit flash ADC, the analog voltage is fed simultaneously to $2^N - 1$ comparators. The output of the comparators is then encoded to a binary format using digital circuit. Assume that the analog voltage source V_{in} (whose output is being converted to digital format) has a source resistance of 75Ω as shown in the circuit diagram below and the input capacitance of each comparator is 8 pF. The input must settle to an accuracy of ½ LSB even for a full scale input change for proper conversion. Assume that the time taken by the thermometer to binary encoder is negligible.

If the flash ADC has 8 bit resolution, which one of the following alternatives is closest to the maximum sampling rate?
(A) 1 megasamples per second
(B) 6 megasamples per second
(C) 64 megasamples per second
(D) 256 megasamples per second

43. Ans: (B)

Sol:

$$V_{in}^1 = \frac{V_{in}}{RC_{eq}} T$$

V_{in}^1 has to settle down within $\frac{1}{2}$ LSB of full scale value.

i.e $\dfrac{509}{510} V_{in} = \dfrac{V_{in.} T}{75 \times (255 \times 8 \times 10^{-12})} \Rightarrow T = (75 \times 255 \times 8 \times 10^{-2}) \times \dfrac{509}{510}$

$$T \approx 0.15 \ \mu sec$$

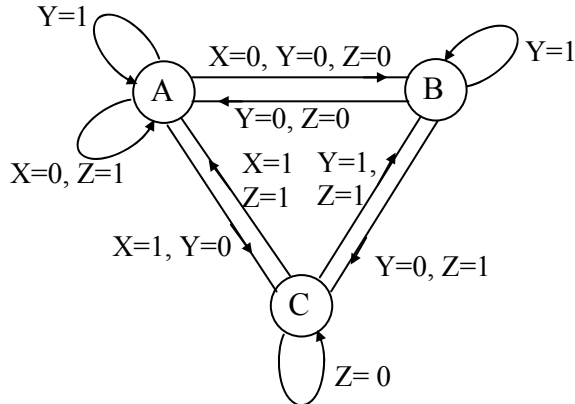Thus sample period $T_s \geq T$

$$T_s \geq 0.15 \ m \ sec$$

$$f_s, max = \frac{1}{Ts_{,min}} = \frac{1}{0.15 \times 10^{-6}} Hz \approx 6 \ Megasamples$$

44. The state transition diagram for a finite state machine with states A, B and C, and binary input X, Y and Z, is shown in the figure.

Which one of the following statements is correct?
(A) Transitions from State A are ambiguously defined
(B) Transition from State B are ambiguously defined
(C) Transitions from State C are ambiguously defined
(D) All of the state transitions are defined unambiguously.

44.   Ans: (C)

Sol: In state (C), when XYZ = 111, then Ambiguity occurs
Because, from state (C) $\Rightarrow$ When X = 1, Z = 1 $\Rightarrow$ N.S is (A)
                                      When Y = 1, Z = 1 $\Rightarrow$ N.S is (B)

45. In the feedback system shown below $G(S) = \dfrac{1}{(s^2 + 2s)}$.

The step response of the closed-loop system should have minimum setting time and have no overshoot.

The required value of gain k to achieve this is _____

45. Ans: 1

Sol: Given $G(s) = \dfrac{1}{(s^2 + 2s)}$

From Diagram CE $\Rightarrow 1 + KG(s) = 0$

$s^2 + 2s + K = 0$

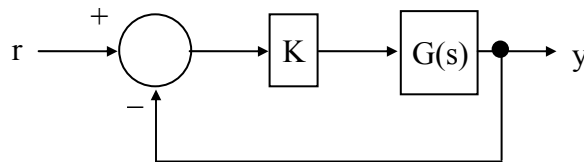Minimum Settling Time is obtain. For Critical Damped System For Critical Damped System $(\xi = 1)$ the % $m_p = 0\%$

$2\xi\omega_n = 2$

$2 \times 1 \times \omega_n = 2$

$\omega_n = 1$ rad/sec

$K = 1$

46. In the feedback system shown below $G(S) = \dfrac{1}{(s+1)(s+2)(s+3)}$

The positive value of k for which the gain margin of the loop is exactly 0 dB and the phase margin of the loop is exactly zero degree is _____

46. Ans: 60

Sol: Given Forward path TF $= \dfrac{1}{(s+1)(s+2)(s+3)}$

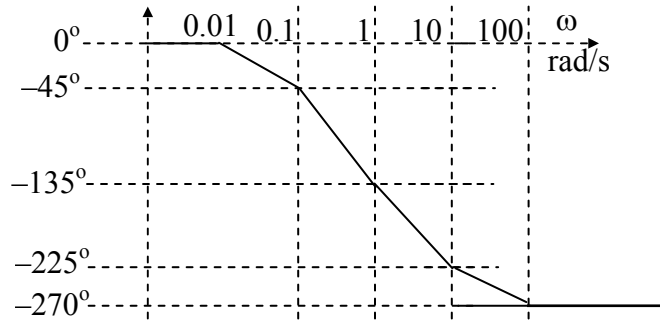Given GM = 0dB, PM = 0^0 That Means Given System is Marginal Stable

$1+KG(s) = 0 \Rightarrow CE = s^3 + 11s^2 + 6s + 6 + K = 0$

$$
\begin{array}{c|cc}
S^3 & 1 & 6 \\
S^2 & 11 & 6+K \\
S^1 & \left(\dfrac{66-6-K}{11}\right)=0 & \\
S^0 & (6+K) &
\end{array}
$$

$\Rightarrow K = 60$ For Marginal Stable

47. The asymptotic Bode phase plot of $G(S) = \dfrac{k}{(s+0.1)(s+10)(s+p_1)}$, with k and p_1 both positive, is

shown below.

The value of p_1 is _____

47. **Ans: 1**

Sol: From the Bode Diagram at $\omega = 1$, the phase Angle is -135°

$-135^{\circ}|_{\omega=1} = -\tan^{-1}\left(\dfrac{\omega}{0.1}\right) - \tan^{-1}\left(\dfrac{\omega}{10}\right) - \tan^{-1}\left(\dfrac{\omega}{p_1}\right)$

$-135^{\circ} = -\tan^{-1}\left(\dfrac{1}{0.1}\right) - \tan^{-1}\left(\dfrac{1}{10}\right) - \tan^{-1}\left(\dfrac{1}{P_1}\right)$

$-135^{\circ} = -84.28 - 5.71 - \tan^{-1}\left(\dfrac{1}{P_1}\right)$

$45^{\circ} = \tan^{-1}\left(\dfrac{1}{p_1}\right) \Rightarrow 1 = \dfrac{1}{P_1} \Rightarrow P_1 = 1$

48. An information source generates a binary sequence $\{\alpha_n\}$. α_n can take one of the two possible values -1 and $+1$ with equal probability and are statistically independent and identically distributed. This sequence is pre-coded to obtain another sequence $\{\beta_n\}$, as $\beta_n = \alpha_n + k\,\alpha_{n-3}$. The sequence $\{\beta_n\}$ is used to modulate a pulse g(t) to generate the baseband signal

$$X(t) = \sum_{n=-\infty}^{\infty}\beta_n g(t-nT),\ \text{where}\ g(t) = \begin{cases} 1, & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

If there is a null at $f = \dfrac{1}{3T}$ in the power spectral density of X(t), then k is_____

48. **Ans: –1**

Sol: The PSD of $X(t) = S_X(t)$

$S_X(f) = \dfrac{|G(f)|^2}{T}\left(\sum_{m=-\infty}^{\infty} R_b(\tau)e^{j2\pi f\tau T}\right)$

$R_b(\tau) = E[\beta_n B_{n-\tau}] = E[(\alpha_n + k\alpha_{n-3})(\alpha_{n-\tau} + k\alpha_{n-\tau-3})] = E[\alpha_n \alpha_{n-\tau} + k\alpha_n \alpha_{n-\tau-3} + k\alpha_{n-3}\alpha_{n-\tau} + k^2 \alpha_{n-3} \alpha_{n-\tau-3}]$

$= E[\alpha_n \alpha_{n-\tau}] + kE[\alpha_n \alpha_{n-\tau-3}] + kE[\alpha_{n-3}\alpha_{n-\tau}] + k^2E[\alpha_{n-3} \alpha_{n-\tau-3}]$

$= E[\alpha_n \alpha_{n-\tau}] + kE[\alpha_n \alpha_{n-\tau-3}] + kE[\alpha_{n-3}\alpha_{n-\tau-3+3}] + k^2E[\alpha_{n-3} \alpha_{n-\tau-3}]$

$= R(\tau) + kR(\tau+3) + kR(\tau-3) + k^2 R(\tau) = (1+k^2)R(\tau) + kR(\tau+3) + kR(\tau-3)$

The Auto correlation function is

$$R_b(\tau) = \begin{cases} 1+k^2 & \tau = 0 \\ k & \tau = \pm 3 \\ 0 & \text{otherwise} \end{cases}$$

Power spectral density

$S_b(f) = 1 + k^2 + 2k \cos(2\pi f 3T)$

Null will occur at $f = \dfrac{1}{3T}$

So at $f = \dfrac{1}{3T} \Rightarrow S_b(f) = 1 + k^2 + 2k \cos 2\pi \left(\dfrac{1}{3T}\right) \times 3T = 0$

$\Rightarrow 1 + k^2 + 2k = 0 \Rightarrow (k+1)^2 = 0 \Rightarrow k = -1$

49. An ideal band-pass channel 500 Hz-2000 Hz is deployed for communication. A modem is designed to transmit bits at the rate of 4800 bits/s using 16-QAM. The roll-off factor of a pulse with a raised cosine spectrum that utilizes the entire frequency band is_____

49. Ans: 0.25

Sol: Bw = 1500 Hz

$$\frac{R_b[1+\alpha]}{\log_2^{16}} = 1500Hz$$

$$\Rightarrow R_b[1+\alpha] = 1500 \times 4 = 6000 \Rightarrow (1+\alpha) = \frac{6000}{4800}$$

$$\Rightarrow \alpha = \frac{6000}{4800} - 1 = 0.25$$

50. Consider random process $X(t) = 3V(t) - 8$, where V (t) is a zero mean stationary random process with autocorrelation $R_v(\tau) = 4e^{-5|\tau|}$. The power is X(t) is_____

50. Ans: 100 W

Sol: $E[x^2(t)] = E[(3V(t) - 8)^2] = E[(3V(t)^2 + 64 - 2 \times 3V(t) \times 8]$
$= E[(9V^2(t) + 64 - 48V(t)] = 9E[V^2(t)] + E[64] - 48\ E[V(t)]^{\,0}$

$$\because [E[V(t)] = 0, \quad E[V^2(t)] = MS = R(0) = 4e^{-5(0)} = 4, \quad E[\text{constant}] = \text{constant}]$$

$E[x^2(t)] = 9 \times 4 + 64 = 36 + 64 = 100$

51. A binary communication system makes use of the symbols "zero" and "one". There are channel errors. Consider the following events:

  x_0 : a "zero" is transmitted
  x_1 : a "one" is transmitted
  y_o : a " zero" is received
  y_1: a " one" is received

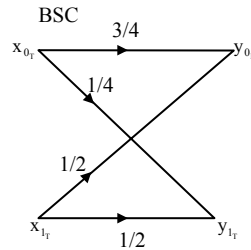The following probabilities are given: $P(x_o) = \dfrac{1}{2}, P(y_o | x_o) = \dfrac{3}{4}, \text{and } P(y_o | x_1) = \dfrac{1}{2}.$ The information in bits that you obtain when you learn which symbol has been received (while you know that a "zero" has been transmitted) is _____
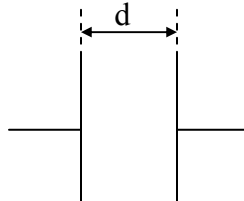
51.   Ans: 0.4056

Sol:

P(y/x)	y_0	y_1
x_0	3/4	1/4
x_1	1/2	1/2

P(xy)	y_0	y_1
x_0	3/8	1/8
x_1	1/4	1/4

$$H(y/x_o) = P(x_o y_o)\log_2\left(\frac{1}{P(y_0/x_0)}\right) + P(x_o y_1)\log_2\left(\frac{1}{P(y_1/x_0)}\right)$$

$$= -P(x_0 y_0).\log_2 P(y_0/x_0) - P(x_0 y_1).\log_2 P(y_1/x_0)$$

$$= -\frac{3}{8}\log_2\frac{3}{4} - \frac{1}{8}\log_2\frac{1}{4} = 0.4056$$

52.   The parallel-plate capacitor shown in the figure has movable plates. The capacitor is charged so that the energy stored in it is E when the plate separation is d. The capacitor is then isolated electrically and the plates are moved such that the plate separation become 2d.

At this new plate separation, what is the energy stored in the capacitor, neglecting fringing effects?

(A) 2E   (B) $\sqrt{2}$ E   (C) E   (D) E/2

52.   Ans: (A)

Sol: Energy stored when spacing is d is given by
Energy stored = Energy density × volume
$E_1 = E_d \times V_1$
$V_1 = d_1 A = dA$
When spacing between the plated is doubled, $d_2 = 2d$
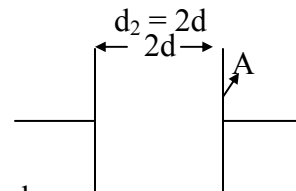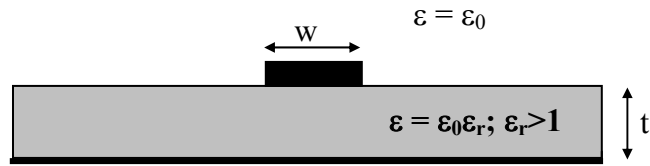Then, $V_2 = d_2 A = 2dA$
$E_2 = E_d \times 2dA$
$ = 2E_d (dA)$
$E_2 = 2E_1$
There with the modified capacitor energy stored is doubled.

53.   A lossless microstrip transmission line consists of a trace of width w. It is drawn over a practically infinite ground plane and is separated by a dielectric slab of thickness t and relative permittivity $\varepsilon_r > 1$. The inductance per unit length and the characteristic impedance of this line are L and Z_o, respectively.
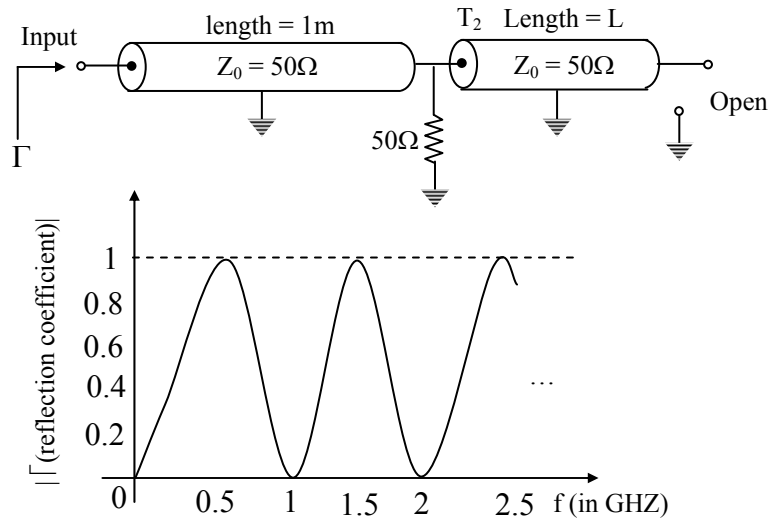
$$\varepsilon = \varepsilon_0$$

w

$$\varepsilon = \varepsilon_0\varepsilon_r; \ \varepsilon_r > 1$$ t

Which one of the following inequalities is always satisfied?`

(A) $Z_o > \sqrt{\dfrac{Lt}{\varepsilon_o\varepsilon_r w}}$ (B) $Z_o < \sqrt{\dfrac{Lt}{\varepsilon_o\varepsilon_r w}}$ (C) $Z_o > \sqrt{\dfrac{Lw}{\varepsilon_o\varepsilon_r t}}$ (D) $Z_o < \sqrt{\dfrac{Lw}{\varepsilon_o\varepsilon_r t}}$
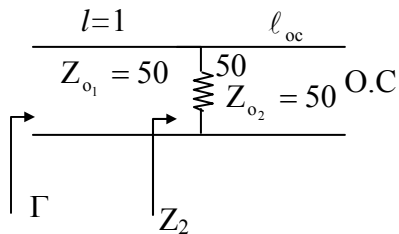
53.   Ans: (A)

54.   A microwave circuit consisting of lossless transmission lines T_1 and T_2 is shown in the figure. The plot shows the magnitude of the input reflection coefficient Γ as a function of frequency f. The phase velocity of the signal is transmission lines is 2×10^8 m/s.

54.   Ans: 0.1
Sol:

$Z_L = 50 \ // -j \ 50 \cot \beta \ell_{o_c}$

$|\Gamma| = \left|\dfrac{Z_2 - Z_{01}}{Z_2 + Z_0}\right| = 0$ only when $Z_L = Z_{o_1}$

$50 \ // -j \ 50 \cot \beta \ l_{oc} = 50$

This satisfied only when $-j\,50\,\cot\beta\,l_{oc} = \infty$

i.e., $\beta\,l_{oc} = m\pi$

$$\frac{2\pi}{\lambda}\ell_{oc} = m\pi$$

$$\ell_{oc} = \frac{m\lambda}{2}$$

$$\ell_{oc} = \frac{mv}{2f}$$

$$= \frac{m\times2\times10^{8}}{2\times f\times10^{9}} \quad [\because \text{f in GHZ ,Here f = 0,1,2,3.. GHZ}]$$

$$\ell_{oc} = \frac{m}{10f}$$
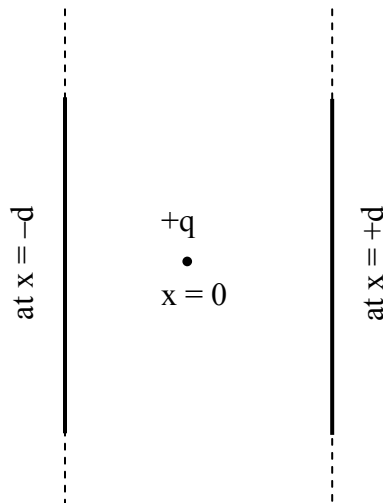
f = 1GHz (Here f= 1 GHz m = 1 for minimum length l_{oc})

$$\ell_{oc} = \frac{m}{10}$$

$$\ell_{oc} = \frac{1}{10} \quad [\text{for m = 1}]$$

$L_{oc} = 0.1$

55. A positive charge q is placed at x = 0 between two infinite metal plates placed at x = −d and at x = +d respectively. The metal plates lie in the yz plane.

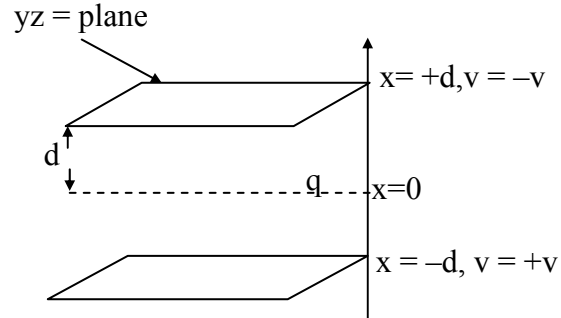The charge is at rest at t = 0, when a voltage +V is applied to the plate at −d and voltage −V is applied to the plate at x = +d. Assume that the quantity of the charge q is small enough that it does not perturb the field set up by the metal plates. The time that the charge q takes to reach the right plate is proportional to

(A) d/V   (B) $\sqrt{d}$ /V   (C) d/$\sqrt{V}$ (D) $\sqrt{d/V}$

55.   Ans: (C)

Sol:

Fig

When there is no external field,

Change at rest having potential energy only

P.E = qv

By an application of an external field, change carries acquire some kinetic energy, with velocity V.

$qv = \frac{1}{2} mv^2$

$V = \sqrt{\dfrac{2eV}{m}}$

Time taken to reach x = d plate is known as g_{tr} 'Gap transit' time

$t_g = \dfrac{d}{v} = \dfrac{d}{\sqrt{\dfrac{2eV}{m}}}$

$\therefore t_d \alpha \dfrac{d}{\sqrt{V}}$