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H.O: 204, II Floor, Rahman Plaza, Opp. Methodist School, Abids, Hyderabad-500001,

Ph: 040-23234418, 040-23234419, 040-23234420, 040 - 24750437

ESE- 2018 (Prelims) - Offline Test Series

Test-20

GENERAL STUDIES

SUBJECT: ENGINEERING MATHEMATICS AND NUMERICAL ANALYSIS SOLUTIONS

01. Ans: (b)

$$\begin{aligned} \text{Sol: } \int_0^{2\pi} |\cos x| dx &= 2 \int_0^{\pi} |\cos x| dx = 4 \int_0^{\frac{\pi}{2}} |\cos x| dx \\ &= 4 \int_0^{\frac{\pi}{2}} (\cos x) dx = 4(\sin x)_0^{\frac{\pi}{2}} \\ &= 4(1 - 0) = 4 \end{aligned}$$

02. Ans: (a)

$$\begin{aligned} \text{Sol: } \lim_{x \rightarrow 2^-} f(x) &= \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2} f(x) = 3 \\ \therefore \text{The above function is continuous at } x &= 2 \\ \text{but } f'(x) &= \begin{cases} 1, & x \leq 2 \\ -1, & x > 2 \end{cases} \end{aligned}$$

$$\begin{aligned} \therefore \lim_{x \rightarrow 2^-} f'(x) &= 1 \text{ and } \lim_{x \rightarrow 2^+} f'(x) = -1 \\ \text{i.e. } f(x) &\text{ is not differentiable at } x = 2 \end{aligned}$$

03. Ans: (c)

$$\begin{aligned} \text{Sol: } f'(x) &= 3x^3 + 24x^2 + 45x \\ f'(x) = 0 &\Rightarrow 3x(x^2 + 8x + 15) = 0 \\ &\Rightarrow 3x(x + 3)(x + 5) = 0 \\ \therefore x = 0, -3, -5 &\text{ are stationary points} \\ f''(x) &= 3(3x^2 + 16x + 15) \\ \text{at } x = 0, f''(0) &= 45 > 0 \\ \therefore f(x) &\text{ has a minimum at } x = 0 \end{aligned}$$

$$\begin{aligned} \text{at } x = -3, f''(-3) &= -18 < 0 \\ \therefore f(x) &\text{ has a maximum at } x = -3 \end{aligned}$$

$$\begin{aligned} \text{at } x = -5, f''(-5) &= 30 > 0 \\ \therefore f(x) &\text{ has a minimum at } x = -5 \end{aligned}$$

04. Ans: (b)

$$\begin{aligned} \text{Sol: } \lim_{x \rightarrow 0} \frac{x(1 - a \cos x) + b \sin x}{x^3} &= \frac{1}{3} \\ \Rightarrow \frac{1}{3} &= \lim_{x \rightarrow 0} \frac{(1 - a \cos x) + a x \sin x + b \cos x}{3x^2} \\ &\text{(Applying L-Hospital's rule) (1)} \\ \Rightarrow (1 - a + b) &= 0 \text{ (2)} \\ &\text{(: Limiting value is finite)} \end{aligned}$$

Again by applying L-Hospital's rule two times,

$$\begin{aligned} \frac{1}{3} &= \lim_{x \rightarrow 0} \frac{a \sin x + a(\sin x + x \cos x) - b \sin x}{6x} \\ &= \lim_{x \rightarrow 0} \frac{a \cos x + a(\cos x + x \sin x + \cos x) - b \sin x}{6} \\ &= \lim_{x \rightarrow 0} \frac{3a \cos x - a x \sin x - b \cos x}{6} \\ &= \frac{3a - b}{6} \\ \therefore (3a - b) &= 2 \text{ (3)} \\ \text{Solving (2) \& (3) for } a, b; &\text{ we get,} \\ a = \frac{1}{2} \text{ \& } b = \frac{-1}{2} \end{aligned}$$

Pre GATE-2018

COMPUTER BASED TEST

Date of Exam : 20th Jan 2018

Last Date To Apply : 05th Jan 2018

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05. Ans: (d)

Sol: By Green's theorem,

$$\begin{aligned} & \int_C (x^2 + xy)dx + (x^2 + y^2)dy \\ &= \int_{-1}^1 \int_{-1}^1 \left[\frac{\partial}{\partial x}(x^2 + y^2) - \frac{\partial}{\partial y}(x^2 + xy) \right] dx dy \\ &= \int_{-1}^1 \int_{-1}^1 (2x - x) dx dy \\ &= \int_{-1}^1 x dx \int_{-1}^1 dy \\ &= \left(\frac{x^2}{2} \right)_{-1}^1 (y)_{-1}^1 = 0 \end{aligned}$$

06. Ans: (c)

Sol: Let $\phi(x, y, z) = (x^2 + y^2 + z^2)$

$$\nabla\phi = (2x\bar{i} + 2y\bar{j} + 2z\bar{k})$$

$$(\nabla\phi)_{\text{at } (1, 1, 1)} = (2\bar{i} + 2\bar{j} + 2\bar{k}) = 2(\bar{i} + \bar{j} + \bar{k})$$

A unit vector in the given direction

$$= \bar{a} = \frac{(\bar{i} + \bar{j} + \bar{k})}{\sqrt{1+1+1}} = \frac{1}{\sqrt{3}}(\bar{i} + \bar{j} + \bar{k})$$

∴ The required directional derivative

$$= (\nabla\phi \cdot \bar{a})$$

$$= \frac{2}{\sqrt{3}}(1+1+1)$$

$$= \frac{2}{\sqrt{3}}(3) = 2\sqrt{3}$$

07. Ans: (d)

Sol: The auxiliary equation is $D^2 + 25 = 0$

$$\Rightarrow D = \pm 5i$$

$$y = C_1 \cos 5x + C_2 \sin 5x \dots\dots\dots (1)$$

$$\text{at } x = 0, y = 1 \Rightarrow 1 = C_1 + 0$$

$$\text{at } x = \pi, y = -1 \Rightarrow -1 = C_1(-1) + 0$$

$$\Rightarrow C_1 = 1$$

substituting C_1 value in (1), we have

$$y = \cos 5x + C_2 \sin 5x$$



08. Ans: (d)

Sol: If the partial differential equation

$$A \frac{\partial^2 p}{\partial x^2} + B \frac{\partial^2 p}{\partial x \partial y} + C \frac{\partial^2 p}{\partial y^2} + D \frac{\partial p}{\partial x} + E \frac{\partial p}{\partial y} = 0$$

is parabolic,

then $B^2 - 4AC = 0$.

Here, $A = 1$, $B = 4$ and $C = K$

$$16 - 4 \times 1 \times K = 0 \Rightarrow 4K = 16 \\ \Rightarrow K = 4$$

09. Ans: (b)

Sol: $4x^2 \frac{d^2 y}{dx^2} + y = 0$

Let $x = e^t \Rightarrow \log x = t$

$$4 D_1(D_1 - 1)y + y = 0 \quad (\text{where } D_1 = \frac{d}{dt})$$

$$(4D_1^2 - 4D_1 + 1)y = 0$$

The auxiliary equation is

$$4D_1^2 - 4D_1 + 1 = 0$$

$$\Rightarrow (2D_1 - 1)^2 = 0 \Rightarrow D_1 = \frac{1}{2}, \frac{1}{2}$$

$$y = (C_1 + C_2 t) e^{\frac{t}{2}}$$

$$y = (C_1 + C_2 \log x) x^{\frac{1}{2}}$$

10. Ans: (c)

Sol: $\frac{d^2 y}{dt^2} - 16y = e^{-4t} - \sin 4t$

$$P. I. = \left(\frac{1}{D^2 - 16} \right) (e^{-4t} - \sin 4t)$$

$$= t \left(\frac{1}{2D} \right) e^{-4t} - \frac{1}{-16 - 16} \sin 4t$$

$$= \frac{t}{-8} e^{-4t} + \frac{1}{32} \sin 4t$$

$$= -\frac{t}{8} e^{-4t} + \frac{\sin 4t}{32}$$

11. Ans: (d)

Sol: $\frac{dy}{dx} + 1 = e^{x+y}$

$$\text{Let } x + y = v \Rightarrow 1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

$$\frac{dv}{dx} - 1 + 1 = e^v$$

$$\frac{dv}{dx} = e^v$$

$$e^{-v} dv = dx$$

$$-e^{-v} = x + c$$

$$x + e^{-(x+y)} = -C$$

or
 $x + e^{-(x+y)} = A$

12. Ans: (a)

Sol: $(x^2 + y^2 + 1)dx - 2xy dy = 0$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{2y + 2y}{-2xy} = \frac{4y}{-2xy} = \frac{-2}{x}$$

$$I. F = e^{\int \frac{-2}{x} dx} = e^{-2 \log x} = x^{-2}$$

$$I. F = \frac{1}{x^2}$$

13. Ans: (a)

Sol: $P.I = \left(\frac{1}{D^2 + D - 6} \right) x = \frac{1}{-6 \left[1 - \left(\frac{D^2 + D}{6} \right) \right]}$

$$= -\frac{1}{6} \left[1 - \left(\frac{D^2 + D}{6} \right) \right]^{-1} x$$

$$= -\frac{1}{6} \left[1 + \left(\frac{D^2 + D}{6} \right) + \dots \right] x$$

$$= -\frac{1}{6} \left(x + \frac{1}{6} \right) = -\frac{1}{36} (6x + 1)$$



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14. Ans: (c)

Sol: $\frac{dy}{dx} + \frac{y}{x} = x^2$

Integrating factor = $e^{\int \frac{1}{x} dx} = e^{\log x} = x$

The solution is

$$y \cdot x = \int x^2 \cdot x dx + C$$

$$xy = \frac{x^4}{4} + C$$

$$\text{at } x = 1, y = 1 \Rightarrow 1 = \frac{1}{4} + C \Rightarrow C = \frac{3}{4}$$

$$xy = \frac{x^4}{4} + \frac{3}{4}$$

$$4xy = x^4 + 3$$

15. Ans: (c)

Sol: A. $E = D^4 + 3D^2 = 0$

$$D^2(D^2 + 3) = 0$$

$$D = 0, 0, D = \pm \sqrt{3}i$$

$$x = C_1 + C_2 t + C_3 \cos \sqrt{3} t + C_4 \sin \sqrt{3} t$$

16. Ans: (a)

Sol: $u = 2xy$

By C-R equations,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \& \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$dv = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy = \frac{-\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy$$

$$\therefore v = \int_{(y \text{ constant})} \frac{-\partial u}{\partial y} dx + \int_{(\text{free from } x)} \frac{\partial u}{\partial x} dy + \text{Constant}$$

$$= \int -2x dx + \int 2y dy + \text{Constant}$$

$$= -x^2 + y^2 + \text{Constant}$$

17. Ans: (b)

Sol: for singularities

$$\text{let } (z-1)(z-2) = 0 \Rightarrow z = 1, 2$$

for zeros

$$\text{let } z + 2 = 0 \Rightarrow z = -2$$

GATE - 2018
ONLINE TEST SERIES

No. of Tests : 62

All tests will be available till
12th February 2018

ESE - 2018 PRELIMS
ONLINE TEST SERIES

No. of Tests : 44

All tests will be available till
07th January 2018

ISRO - 2017
ONLINE TEST SERIES

No. of Tests : 15

All tests will be available till
25th December 2017

★ HIGHLIGHTS ★

- ⊗ Detailed solutions are available.
- ⊗ **All India rank** will be given for each test.
- ⊗ Comparison with all India toppers of **ACE** students.

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25. Ans: (c)

$$\text{Sol: Combined } \bar{X} = \frac{n_1 \bar{X}_1 + n_2 \bar{X}_2 + n_3 \bar{X}_3}{n_1 + n_2 + n_3}$$

$$\bar{X}_2 = \frac{300 \times 45 - 100 \times 70 - 100 \times 20}{100} = \frac{4500}{100} = 45$$

26. Ans: (c)

$$\text{Sol: } \sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$= \frac{169}{10} - \left(\frac{12}{10}\right)^2 = 1.69 - (1.2)^2 = 0.25$$

27. Ans: (b)

Sol: For sum to be 7: {(1, 6) (6, 1) (5, 2) (2, 5) (4, 3) (3, 4)}

Number of favourable cases = 2

$$\text{Required probability} = \frac{2}{6}$$

28. Ans: (d)

Sol:

$$\begin{aligned} \text{Required Probability} &= 1 - P(5 \cup 6) \\ &= 1 - \left[\frac{6}{30} + \frac{5}{30} - \frac{1}{30} \right] \\ &= \frac{20}{30} \end{aligned}$$

29. Ans: (a)

Sol: $n(S) = 2^5$

$$\begin{aligned} \text{Required probability} &= P(2 \text{ heads and } 3 \text{ tails}) \\ &\quad + P(3 \text{ heads and } 2 \text{ tails}) \\ &= \frac{{}^5C_2 + {}^5C_3}{2^5} = \frac{10 + 10}{2^5} \\ &= \frac{20}{2^5} \end{aligned}$$

30. Ans: (a)

Sol: Required probability

$$\begin{aligned} &= P(A \cup B) - P(A \cap B) \\ &= \frac{11}{20} - \frac{10}{20} = \frac{1}{20} \end{aligned}$$



31. Ans: (a)

Sol: Given that, $P(M \cup S) = \frac{7}{10}$;

$$P(M \cap S) = \frac{2}{5}; P(M/S) = \frac{2}{3}$$

Required probability = $1 - P(S)$

$$= 1 - \left(\frac{P(M \cap S)}{P(M/S)} \right) = 1 - \frac{2/5}{2/3} = \frac{2}{5}$$

$$\left(\because P(M/S) = \frac{P(M \cap S)}{P(S)} \right)$$

32. Ans: (a)

Sol: $E(X) = (0 \times P) + (2P) + 1(1 - 2P) = 1$

33. Ans: (b)

Sol: $E(X^2) = \int_{-4}^4 x^2 |x| dx$

$$= 2 \int_0^4 x^3 dx$$

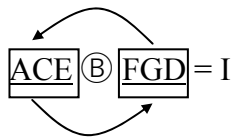
$$= 2 \left[\frac{x^4}{4} \right]_0^4 = 2 \left[\frac{4^4}{4} \right]$$

$$= 2[4^3] = 128$$

34. Ans: (c)

Sol:

Short cut solution:



$$B^{-1} = FGDAEC$$

Solution: Given that $ACEBFGD = I$

$$\Rightarrow ACEB = D^{-1} G^{-1} F^{-1}$$

$$\Rightarrow B = E^{-1} C^{-1} A^{-1} D^{-1} G^{-1} F^{-1}$$

$$\Rightarrow B^{-1} = FGDAEC$$

35. Ans: (b)

Sol: $A^2 = I$

$$|A^2| = |I|$$

$$|A^2| = 1$$

$$|A| = \pm 1$$

$$\therefore |A| \neq 0$$

$\Rightarrow A$ is non-singular matrix

$$\therefore \rho(A) = n \text{ and } \rho(A|B) = n$$

$$\rho(A) = \rho(A|B) = \text{number of unknowns}$$

\therefore The system has a unique solution.

36. Ans: (b)

Sol: $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 0$

$$\rho(A) < 3 \text{ (Number of unknowns)}$$

\therefore It has non-trivial solution

$$\text{Rank of } A = 2$$

\Rightarrow The system has only one independent solution

37. Ans: (c)

Sol: Number of Linear independent solutions

$$= n - r = n - \rho(A)$$

$$= 4 - 1 = 3$$

$$A = \begin{bmatrix} 1 & 2 & 3 & -2 \\ 3 & 6 & 9 & -6 \\ 2 & 4 & 6 & -4 \end{bmatrix}$$

$$R_1 - 3R_1, R_3 - 2R_1$$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \rho(A) = 1$$



38. Ans: (b)

Sol: Trace $A = 2 + y$

$$4 + 8 = 2 + y$$

$$y = 10$$

$$|A| = 2y - 3x$$

$$4 \times 8 = 2y - 3x$$

$$32 = 20 - 3x$$

$$\Rightarrow x = -4$$

39. Ans: (d)

Sol: Given that eigen values of A are $1, -1, 0$

The eigen values of $(A^{25} + 5I)$ are $1^{25} + 5,$
 $(-1)^{25} + 5, 0^{25} + 5$

i.e. $6, 4$ and 5

$$|A^{25} + 5I| = 4 \times 5 \times 6 = 120$$

40. Ans: (c)

Sol: Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$AX = \lambda X$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{aligned} a - b &= 1 \\ c - d &= -1 \end{aligned}$$

$$AX = \lambda X$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 4 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{aligned} 2a + b &= 8 \\ 2c + d &= 4 \end{aligned}$$

$$\begin{aligned} a &= 3, b = 2 \\ c &= 1, d = 2 \end{aligned}$$

41. Ans: (d)

Sol: Number of Linear Independent eigen vectors

$$= n - \rho(A - \lambda I) \quad \lambda = 100$$

$$= 4 - \rho(A - 100I)$$

$$= 4 - 0$$

$$= 4$$

42. Ans: (b)

Sol:

$$(A + I)^{-1} (A + 5I) = (A + I)^{-1} [(A + I) + 4I]$$

$$= [(A + I)^{-1} (A + I) + 4(A + I)^{-1}]$$

$$(A + I)^{-1} (A + 5I) = I + 4(A + I)^{-1}$$

Given that, eigen values of A are -2 and -3 .

\therefore Eigen values of $[I + 4(A + I)^{-1}]$ are

$$1 + 4(-2 + 1)^{-1}, 1 + 4(-3 + 1)^{-1}$$

i.e. -3 and -1

43. Ans: (a)

Sol: Error = 0

Simpson $\frac{1}{3}$ gives exact result for polynomial of degree 2.

44. Ans: (a)

45. Ans: (b)

Sol: The Newton- Raphson iteration formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{where } f(x) = x^2 - 2$$

$$\Rightarrow x_{n+1} = \frac{x_n^2 + 2}{2x_n}$$

$$x_1 = \frac{x_0^2 + 2}{2x_0} = \frac{(1)^2 + 2}{2(1)} = 1.5$$

46. Ans: (b)



47. Ans: (c)

Sol: In Newton Raphson method, if the graph of $f(x)$ is horizontal to x-axis then we cannot draw tangent. So we cannot find better approximation.

48. Ans: (c)

Sol: If $f(a).f(b) > 0$ then we have no root (or) even number of roots.

49. Ans: (b)

Sol: In Newton Raphson method, we have $e_{n+1} = k e_n^2$ so, it has Quadratic convergence

50. Ans: (a)

GATE TOPPERS

GATE 2017

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							10 IN RISHI SHYAMRAO

ESE TOPPERS

ESE 2017

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